ABSTRACT: The aim of this paper is to reconsider the controversial problem of the relationship between the philosophy of Hegel and Leibniz. Beyond the thick curtain of historical references (which have been widely developed by scholars), it is in fact possible to assume some guideline concepts (i.e. those of ‘singularity’ and ‘infinity’) to reconstruct the deep theoretical influence which Leibniz played in Hegel’s thought since the Jenaer Systementwurf of 1804/05.

KEYWORDS: Leibniz, Hegel, Singularity, Infinite.

1. Introduction

In the following pages I will try to suggest some remarks on the concepts of ‘singularity’ and ‘infinite’ in Leibniz and Hegel.

Moving from a brief analysis of the two words, I intend to show how it is possible to find a world of semantic connections that bind Hegel and Leibniz together. Choosing to put ‘singularity’ in relation to the concept of ‘infinite’ was drawn by Hegel’s notation that in Leibniz’s ontology singularity “geht in seiner eigenen Totalität zu Grunde.”¹ Hegel’s somewhat peremptory critique could be seen as a warning that indicates the existence of a problem.

What does Hegel mean when he writes of this Leibnitian bad thematization of ‘singularity?’ What does this concept refer to? And above all, what is the meaning of speaking of a ‘totality’ in reference to a ‘sinking’ singularity? Is it pleonastic or, again, is it a warning that needs to be further analyzed?

From these few and feeble hints, scholars have begun to put the soundness of this general hypothesis to the test. From a semantic point of view, this is like saying that, at the very heart of Modern thought, the constellations of meanings that gravitate around the words ‘singularity’ and ‘infinite’ have known shifts, tensions, and also decisive overlapping that nowadays we often tend to forget.

Whilst the concept of ‘infinite’ has been widely recognized to be as one of the most pregnant terms of Modernity, that of ‘singularity’ has, on the contrary, often been overlooked (unlike other concepts such as ‘individuality’ or ‘identity’ – that are, somehow, related to it).

This is probably due to the latter’s volatility in comparison to the former and also because it has less theoretical appeal. Hegel’s critique of Leibniz’s bad use of ‘singularity’ makes us rethink the very foundations of this concept, and thus, rediscover the strong bond that the two philosophers had built between ‘singularity’ and ‘infinite’ in their respective ontologies.

Both Leibniz and Hegel, indeed, seem to give a certain polarization of such concepts. One cannot deal with either concept without also referring to the other one. Leibniz first, but subsequently Hegel too, believed that a full definition of a singular substance could not be successfully determined without taking into consideration the seemingly opposite notions of ‘totality,’ ‘whole,’ or ‘infinite.’ In the following pages I will try to develop this specific hypothesis.

2. ‘World’ and ‘Singularity: ’Hegel as interpreter of Leibniz

First of all we should try to get a clear picture of general references. Focusing on the project Logik, Metaphisik, Naturphilosophie (Jena 1804/05), let us consider the pages

dedicated to the Monad. As it is known, in JS II we can find the first (and almost complete) part of the system that Hegel drew up in Jena. As far as we know, JS II is also the first work in which he consciously undertakes a theoretical rethinking of Leibniz’s concept of monad and, therefore, is to be considered a fundamental work for our research.

If in Hegel’s previous works the references to Leibniz could seem scarce (for instance, in some of his Stuttgart notes), perhaps only due to the importance of Leibniz’s philosophy in the German philosophical culture at the end of the 18th century (think of the hints in the Dissertatio philosophica de orbis planetarum, or the use of a Leibnitian argument in the polemic against Schulze), in JS II the concept of monad represents a fundamental element in the entire section Metaphysik der Objectivität.3

Having said this, we shall first try to contextualise Hegel’s argumentations. On a broad scale we can say that in this phase of his philosophical development, Hegel thought of logic as an introductory moment to science. Logic has the task of presenting the relations through which the Verstand looks upon its cognitive action, by determining it as its object. This specific type of logic progressively falls apart because of the Verstand’s incapability of recognizing the mutual ‘interrelationships’ among determinations that are, from time to time, by it established. The intellect creates a series of determinations that should be reciprocally independent - the reference here is to Kant and, further still, to 17th century logic school tradition – nonetheless, the increasing contradictions which arise from these static determinations eventually dissolve the work of logic, to the extent that the ‘finiteness’ of the intellectualistic way of thinking emerges from its own dissolving.4

At this point a new ‘metaphysical’ dimension of object emerges, which overcomes the nonsense of the previous intellectual distinctions. ‘Subject’ and ‘object’, ‘spirit’ and ‘world’ are interdependent couples of categories and, thus, they are to be understood on a


metaphysical scale, i.e. assuming that logical determinations and ontological categories involve one another. Metaphysics therefore represents a rational mode (Vernunft), that allows one to observe the very nature of the relation that runs between ‘knowledge’ and its ‘contents.’ In other words, the Vernunft stops regarding its known objects as forms of ‘being-in-itself,’ as something insulated from cognitive processes, because it is rather the contrary: objects are involved in cognition and determine themselves (and their own intelligibility) from within the rational acts.

Therefore, metaphysics can be seen as the development of an immanent articulation of ‘knowledge’ that branches into the following three sections: 1) Das Erkennen als System von Grundsätzen (i.e., the acknowledgement of the very principles that build a cognitive process); 2) the Metaphysik der Objectivität (i.e., the relationship involving knowledge and its ‘objects’); and 3) the Metaphysik der Subjectivität (i.e., the emergence of Subjectivity as the ontological dimension in which the previous oppositions erase each other out).

It is within the second section that we find a hint to Leibnitian monadological principles. Referring to an object as a ‘monad’ Hegel is, in fact, in the condition to overcome the still existing division between ‘knowing’ and known ‘object,’ by considering the latter as a totality. The Monad represents a world and its identity is defined by the totality of relations involved in its individuality; that also means, from Hegel’s point of view, that in a monadic-object every form of ‘otherness’ is absorbed ab origine in its internal development. Far from the Cartesian idea of ‘ob-iectum’ as something distinct and separated from the subject, the Monad is seen by Hegel as the expression of a representative activity in which the subjective side of knowledge and the objective side of content are mutually involved. By using this concept, Hegel can thus offer a broader notion of ‘object’ by including the knowing-subject as part of its structure. As a consequence, we have a strong semantic transformation of the meaning of the word ‘subject,’ which turns out to be not a specific particularity (a ‘this’-individual), but a kind of ontological structure that ultimately unfolds itself as a representation of a world and vice-versa. Object and knowing-subject tie their destiny shifting their meanings.

Let us now go deeper into this. The first element of otherness with which knowledge relates to within the Metaphysics of Objectivity is the notion of ‘soul.’ In this passage, the Seele is thought by Hegel simply as a substantial unity that is opposed to the outer-world. As a ‘Reflexion in sich selbst’ the soul, however, is capable of ‘interiorizing’ its relationship with

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the world. In fact, the soul is nothing else but the very activity of a entering-into-a-relation and it is also the result of this being-determined by relationships. Nonetheless, the emerging structure of subjectivity cannot be fully unfolded at this stage, because the Seele still remains a singularity that lies next to other singularities and, thus, cannot fully assimilate the always resurgent otherness. The soul limits itself only to reflect the image of a ‘preconceived’ world, which in turn is constituted by the representation of an indefinite multitude of ‘external’ connections. Mutual connections of particular singularities, which forms a ‘Kette von Synthese,’ in which all souls and things are contained, eventually creates a contradiction between the modal dimension of how the soul perceives itself and its ontological way of being (i.e., its actual identity conceived as the outcome of the whole process). The soul, setting aside its indifference to the otherness, discovers itself as a für sichseyende Totalität: a fully developed system of reference. A Monad.

What emerges here is the pure shape of a self-relation. These are the conceptual reasons behind Hegel’s rediscovery of Leibniz’s monadological doctrine. The Monad is determined as a subjective activity in which, to put it in Leibniz’s terms, the Cartesian subject-object distinction is finally overcome. The Monad, in fact, is not a simple representation of a world, but ‘it is’ simply a world. Thus, Hegel can write:

Diß Andere ist ebenso ein sich in sich reflectirendes An sich, welches seine Bestimmtheit durch ein anderes selbst bestimmt, sie hiemit aufhebt, und ideel stetzt; oder eben so eine vorstellende Monade als die Seele. (JS II, p. 144).

Whilst the soul represented a knowing subject in its singular opposition to a foreign world, for Hegel (that here begins to twist Leibniz’s doctrine), the concept of Monad allows one to think of the world itself as a knowing totality, as if it were at the same time a reflective subjective structure: “das in sich reflectirte und sich reflectirende.” The perceptive activity of Monads represents (sich vorstellen) otherness as a part of its developmental inner contents. This self-explaining circle represents the way in which the soul comes to coincide with its own complete concept, the collapsing point in which the epistemological point comes to coincide with the ontological nature of its individuality.

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5 See JS II, p. 142.
6 Ivi, p. 143.
7 The Monad is “schlechtin die Beziehung in sich selbst,” and for this it is the pure form of a ‘totality’ (see JS II, p. 146)
8 Ivi, p. 143.
This is indeed the impact that Monadic philosophy has had on Hegel’s early thought. Nonetheless, Hegel does not stop here and transforms the concept of Monad even further. In Leibniz’s system the Monads were infinite, not only numerically, but also qualitatively, because each of them was considered to be an expression of a world in its infinite individuality. A typically Leibnitian issue consisted indeed in the ‘harmonization’ of infinite Monadic worlds (each of them was the bearer of an infinite interiority). As we have seen, Hegel interprets the Monad not so much as an individual entity, but rather as the pure form of a self-relation. By doing so he combines the epistemological level (the way in which the individual-monad acknowledges itself as a world of infinite interrelationships) with an ontological issue (what the monadic-individuality actually is), whilst according to Leibniz the connection between these two spheres was somehow problematic and not at all self-evident.  

From Hegel’s point of view, the presence of a multiplicity of different self-reflective structures (each maintaining its own singular individuality) is somewhat secondary to the problem of the form of the self-relation itself. He detaches, so to say, this logic-ontological point from the rest, while in Leibniz’s universe to consider the pure shape of a self-relation, independently from the substantial relata, does not make any sense (because of Leibniz’s nominalism).

This is where the two theories are harder to compare. For Leibniz, modal self-relations stand for an essential trait that defines a particular class of individuals (i.e., those endowed with a ‘mind’) and should be considered as an internal distinction among substances. Assuming that every monad entertains a certain relation with its internal states, there are some particular monads that manifest the capacity of reflecting on themselves and of recognizing their psycho-physiological states as the counterparts of the so-called ‘I-thoughts.’ Such particular individuals are called by Leibniz ‘spirits’ and they are thus capable of ‘objectifying’ their inner states. Self-relation is therefore not at all an abstract concept, something that can be

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9 The definition of individual substances, as known, is a complex element in Leibniz’s thought. The relation that binds a ‘monadic individual’ with its ‘complete concept’ is equally a complex thing (more complex than what we can make out from Hegel’s interpretation). The notion of ‘complete concept’ implies a whole series of problems that we will see further on in the following pages. This is to say that it is not prosaic to define what a Monad ‘knows’ of itself and ‘how’ it sees its own representative contents. For further studies on these aspects in Leibniz’s thought see MONDADORI, F. Reference, Essentialism and Modality in Leibniz’s Metaphysics. Studia Leibnitiana, n. 5, 1973, p. 73-101; MATES, B. The Philosophy of Leibniz. Metaphysics and Language. New York/Oxford: Oxford University Press, 1986; ISHIKAWA, H. Leibniz’s Philosophy of Logic and Language. Cambridge: Cambridge University Press, 1990; COVER, J., O’ LEARY-HAWTHORNE, J. Substance and Individuation in Leibniz. Cambridge: Cambridge University Press, 1999; DI BELLA, S. The Science of the Individual: Leibniz’s Ontology of the Individual Substance. Berlin-Dordrecht-Heidelberg-New York: Springer, 2005.
thought of independently from a determined individual mind’s activity, but it is rather bound to an ontological (let us say substantial) core.¹⁰

Conversely, in Hegel’s ontology, at least in this Jena project of metaphysics, the self-relation itself becomes the substantial element of a kind of objective-subjective process. Insofar as the self-relation becomes the only means through which it is possible to define the real nature of ‘substantiality, the strength of the conceptual displacement which invests the very notion of ‘subjectivity’ is also clear. In Hegel’s view, a substance is not an essence to which an infinite ensemble of predicates pertains. It is, rather, a general process of conceptual definition, in which what is being focused is not a specific individual, but the shape of a ‘whole.’¹¹

This way of reading JS II helps us to highlight how, for Hegel, the notion of ‘singularity’ (Einzelnheit) is the very bone of contention (realism vs. nominalism) and, again, how it “geht in seiner eigenen Totalität zu Grunde”, when it is considered as a monadic individual.¹² The Monad is a simple ‘this’ that is set as an ‘absolute infinity.’ It is an individual structure that defines itself as an infinite collection of predicates and it is precisely this mutual relation between singularity and infinity that is the crucial point stressed by Hegel. According to him, the monad is incapable of mastering the infinite multiplicity of its underlying determinations and that’s why the singularity geht zu Grunde. The core of Hegel’s arguments can be seen in this ‘bad’ kind of relation that lies between ‘singularity’ and ‘totality’ (the Monad is an expression of a ‘bad infinity’).¹³ Thus, Hegel writes:

Die Monaden als existierende Dinge drükken nur ein und ebendasselbe Allgemeine aus, ihre Vielheit so wie die Bestimmtheit ihrer Bewegung ist das schlechtin Zufällige und das Existierende, in Beziehung auf die Einzelnheit, in der Tat das nur Mögliche. (JS II, p. 148).

In other words, Hegel believes that in Leibniz’s philosophy the categories of ‘singularity,’ ‘totality,’ ‘infinite,’ ‘being/reality,’ and ‘possibility’ do collapse on one another. Each determination is crystallized and therefore Leibniz needs to introduce an extra-systemic

¹¹ The Monad as ‘totality’ becomes the ‘universal’ for Hegel, and this becomes “aus dem Definitum zu seiner Definition” (see JS II, p. 145).
¹² JS II, p. 144.
¹³ Ivi, p. 145.
concept, that of ‘pre-established harmony,’ to solve the contradictions that undermine the consistence of his ontology.\footnote{See HEGEL, G.W.F. Vorlesungen über die Geschichte der Philosophie. hrsg. von G.J.P.J. Bolland, Leiden 1908, p. 937.}

The conclusion emerging from these pages of JS II runs as follows. The way of being of each Monad does not coincide with its own concept. Monads are purely existing ‘objects,’ but the ontological form of their \textit{Existenz} doesn’t match the epistemological dimension of their self-knowledge. The interwoven totality which substantiates their real structure remains always and constantly a step ahead (or behind) their actual ‘individuality.’ Moreover, if this very conceptual dimension of ‘individuality’ emerged, then the Monads would drop their individual point of view and the single monadic individual would be lost, annihilating its individual identity, but at the same time becoming the \textit{aufgehobene} totality of a more complex and resulting process (‘\textit{das Einzelne ist sich zum ganzen prozess in der Totalität geworden}’).\footnote{JS II, p. 147.}

So much then for JS II then.

But if we take a look at Hegel’s mature works, we can easily observe that his interpretation of the whole matter remains mostly unvaried. Opening the third section of the \textit{Logik (B. Das Objekt)} in the Berlin \textit{Encyclopaedia} we read in the first \textit{Anmerkung}:

\footnote{See BONSIEPEN. Hegels Rezeption, p. 57-62.}

Die Definition: das Absolute ist das Object, ist am bestimmtesten in der Leibnizischen Monaden enthalten, welche ein Object aber an sich vorstellend, und zwar die Totalität der Weltvorstellung seyn soll. (\textit{Enz. C, § 194 Anm.}).

Similarly, monadic individuality is taken into account in the first section of the Nuremberg Logic, entitled \textit{Mechanical Object} (in the section dedicated to a concept’s ‘objectivity’). And the same can be said for the Heidelberg encyclopaedic logic, in which the reference to Monad opens the section ‘B’ (‘The Object’), in the third part of the ‘Doctrine of the Concept.’\footnote{See BONSIEPEN. Hegels Rezeption, p. 57-62.}

Hegel came, thus, to the conclusion that the Monad could be interpreted as a helpful tool to understand the very notion of ‘object’ as it was expounded in his mature system. The reason for this conviction refers precisely to the absence of permeation subsisting between individuality (purely existing \textit{being}) and totality (self-developed ‘concept’). The Monad is a
‘being’ that does not fully coincide with its ‘concept’ and, thus, it takes the shape of a contradiction (“vollständig entwickelte Widerspruch”).

3. Leibniz’s notion of ‘singularity’

If we accept what has been said until now, we might attempt a more careful lexical analysis.

If until now ‘singularity’ has been examined as one of the key concepts of Hegel’s interpretation of Leibniz, we should nevertheless be reminded that this concept has a multiplicity of meanings also in Leibniz’s philosophy.

In the De mundo praesenti, written between 1684 and 1686, when describing the relation between ‘substance’, ‘singularity’ and ‘concept’, Leibniz wrote:

Substantiae autem jam tum concretus est conceptus, nec praedicari potest nisi de substantia, et substantia singularis nulli alii rei singulari inest, cum tamen Accidens singulare substantiae singulari insit, plura accidentia substantiae uni.18

What does Leibniz intend here?

First of all he says: “The concrete element of the substance is the concept” (conceptus). The latter, in its turn: “cannot be predicated, but of substance.” Moreover, we could say, that it is the ‘concept’ that holds and expresses the essential ‘singularity’ of a substance in itself. Leibniz establishes a strong logical relation between ‘concept’ and ‘singularity.’ However, after that, Leibniz states, apparently in a pleonastic way: “substantia singularis nulli alii rei singulari inest.” (AVI 4, B1, p. 1506-1507).

Meaning: ‘singularity’ is not an attribute that can be shared amongst substances. Every substance is expressed by a concept of its own and each ‘singularity’ is irreducible with regards to other existing singularities. In nature two entirely overlapping ‘totalities’ cannot be given and in the very concept of ‘totality’ lies the internal differentiation among individual substances (according to the principle of the identity of indiscernibles).

But there is something more.

The substance’s singularity is expressed by the very concept that defines it, but at the same time the latter represents the collection of those ‘singular accidents’ (predicates) that all together define the singularity of the substantial individuality. Therefore, Leibniz sets a tight relation of inherence between ‘singular accident’ and ‘singular substance,’ where the former cannot be thought, but in relation to a substance, and the latter forms a ‘unity’ only by virtue of the totality of its inhering accidents: “cum tamen Accidens singulare substantiae singulari insit, plura accidentia substantiae uni.” (ibid.)

The terms ‘concept,’ ‘substance,’ ‘singular substance,’ ‘singular accident’ are thus mutually connected. The ‘concept’ is the concrete element of the substance’s singularity. “Conceptus autem completus est nota substantiae singularis.” (ibid.).

But what is a complete concept for Leibniz?

Literally, a complete concept is _qui omnia jam virtute continet, quaecunque de ipso possunt intelligi:_

Et quidem substantiae singularis conceptus est quidam completum, qui omnia jam virtute continet, quaecunque de ipso possunt intelligi. Ita ut Deus eo ipso dum hujus Petri substantiam concipit, omnia concipiat quae ei contingere aut contingent. Quod de accidente singulari dici non potest, nisi quatenus substantiae singulari inesse concipitur. (A VI 4, B, p. 1507).

For instance, when referring to the substance ‘Peter,’ all the determinations that can be referred to Peter will be held in its complete concept, including the contingent ones.19

We must then remember, however, that for Leibniz a singular predicate can be known only because it is part of a ‘totality’ that defines it. The element of ‘singularity’ is such, only when it inheres to the totality of a sequential collection (when it is part of an infinite set of elements). In other words, it can be known – moreover, it can subsist ontologically – only when it is embedded within a conceptual element (to this amounts the doctrine of ‘complete concepts’).20

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20 “Ex Alphonsi regia potestate non concipio ejus Astronomiam, nisi redeam ad ipsius Alphonsi naturam singularem. Porro substantia universalis nihil aliud est quam omnis substantia singularis talis. Cum autem
This is the reason why Leibniz ends the following passage by stating that the ‘universal substance’ is nothing but the very single substance as such. Universality and singularity coincide at the height of the complete concept:

Porro substantia universalis nihil aliud est quam omnis substantia singularis talis. Cum autem substantia singularis involvat relationes ad totum universum patet eam semper esse infinitam. (ibid.)

Insofar the singularity of a substance is nothing but the expression of a totality of relations entertained with the world (the latter perceived by its own perspective), then it is also ‘universal.’

But then again, what is a ‘concept?’ And further still, how does one might one specify the relation between ‘singularity’ and ‘totality?’

Leibniz seems to be telling us two things at the same time:
1- the concept is the ‘concrete’ element of the singular substance, in the sense that without the reference to a totality-system there cannot be any individual singularity.
2- the concept is always the expression of an already existing individual substance, in the sense that if there were not an ontology of substances to support it, the concept alone would not be indicative of anything. The complete concept expresses the singular substance’s specific character, because it takes it to a full manifestation, but it doesn’t ‘create’ anything ex nihilo.21

Therefore, this could be seen as a ‘twofold’ reversible relation, from singular to concept, from concept to singular. Or rather, from singularity to totality and from totality to singularity. Or even, from the universal to the concept and from the concept to the universal. But is it possible to develop all this line of argument, without subsequently transforming everything into a circular reasoning?

Following the classical argument set by the Discourse of Metaphysics, the complete concept holds in itself the totality of possible predicates that inhere to a substance. Namely, it holds in itself the key to a singular individuality. Nonetheless, this can happen only if the concept is ‘complete.’ Only when it is capable of fully expressing a ‘totality.’

Let us try then to investigate the concepts of ‘totality,’ ‘infinite’ and ‘limit’ to see whether we can grasp some helpful pieces of information from this side.

substantia singularis involvat relationes ad totum universum patet eam semper esse infinitam.” See De mundo praesenti, A VI 4, B1, p. 1507.
21 See the character of the concept of ‘note’ given by Leibniz in A VI 4 A, p. 586-587.
3.1. Datis ordinatis etiam quaesita sunt ordinata

When considering Leibniz’s thought about the infinite we must keep in mind that this is, first and foremost, a mathematical issue.\textsuperscript{22} Therefore, at a first glance, the problem of the determination of the infinite can be seen as a focusing of key concepts such as ‘continuum,’ ‘integral,’ ‘momentum,’ ‘evanescent quantities,’ and so on. Let us see how this is done.

First of all, having set the principle of continuum, Leibniz also admits to the presence of some “integral terms” to which “partial terms” can refer to: “Terminum intelligo integralem, nam partiales fiunt ex integrali et particula, ut pars est ens in aliquo” (A VI 4, A2, p. 742)

Thus, for Leibniz, thinking something such as the ‘largest’ or ‘smallest’ of all numbers makes little sense. The notion of infinite becomes comprehensible only if it is switched into a whole/part relation, that is to say only when considering it in a ‘functional way’ (considering the relation subsisting between integral and infinitesimal). In other words, the integral cannot be thought of being the result of a quantitative sum of infinitesimal quantities. Rather, it should be seen as the generative principle of a functional relation: a relation ‘of order’ that stands between integral and infinitesimal.

In a letter sent to Bayle in 1687, Leibniz defines continuity as follows:

Lorsque la difference de deux cas peut estre diminuée au dessus de toute grandeur donnée in datis ou dans ce qui est posé, il faut qu'elle se puisse trouver aussi diminuée au dessus de toute grandeur donné in quaesitis ou dans ce qui en resulte, ou pour parler plus familierement: lorsque le cas (ou ces qui est donné) s'approchent continuellement et se perdent enfin l'un dans l'autre, il faut que les suites ou evenemens (ou ce qui est demandé) le fassent aussi. Ce qui depend encor d'un principe plus general, sçavoir: Datis ordinata etiam quaesita sunt ordinata.\textsuperscript{23}

Leibniz attributed (as Newton did in those years) a qualitative nature to the infinitesimal, and this very same idea will become of paramount importance for Hegel too.

In a later letter to Varignon, Leibniz fully develops this qualitative assumption, as can be seen in the following passage:


il se trouve que les règles du fini réussissent dans l'infini, comme s'il y avait des atomes (c'est à dire des éléments assignables de la nature), quoiqu'il n'y en ait point la matière étant actuellement sousdivisée sans fin; et que vice versa les règles de l'infini réussissent dans le fini, comme s'il y avait des infiniment petits métaphysiques, quoiqu'on n'en ait point besoin; et que la division de la matière ne parvienne jamais à les parcelles infiniment petites.\textsuperscript{24}

Again, what we have here is the idea of a corresponding relation between ‘finite’ and ‘infinite.’ Taking for granted the idea of continuity, infinitesimals must necessarily displace themselves according to that functional order intentioned by the whole.

Let us further consider this fundamental point and let us focus our attention on the third and fundamental element of Leibniz’s mathematical thought: the concept of evanescent quantity. Until now we have seen how Leibniz establishes an essential relation between infinitesimal and integral, meaning that the only way we can relate to the infinitesimals is by setting them in some ‘order,’ similar to the whole/part one. From a mathematical point of view, if we do not take into account this functional relation, it is impossible to solve the problem of a continuum’s determination.

A further step must be taken to get a broader glance of the matter. Once the functionality of the relation that connects infinitesimal and integral is established, we must understand how to concretely develop the qualitative consideration of infinitesimal quantities. For this reason Leibniz used the concept of ‘evanescent quantity,’ that at the time was commonly used in mathematical culture (also by Newton).

We can say that in the procedure of infinitesimal calculus the nature of the evanescent is considered to be both a null and a not-null quantity. This means that when one adds a differential to a finite quantity the result is that very same finite quantity (thus, it acts as a null quantity); whilst, if we consider a relation between differentials (dy/dx), the latter can be, for example, equal to 2, as if dx and dy would both be different from zero. The infinitesimal is therefore not an ‘absolute nothing’ (\textit{nihil absolutum}), but rather, as Leibniz wrote in a letter to the Italian mathematician G. Grandi, a \textit{nihil respectivum} (‘relative nothing’), thereby intending that the variation of quantity functionally depends on the relational context in which it operates.


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This passage is of paramount importance. The evanescent quantity must not be thought as a fixed quantum, but rather as a variable that is functionally linked to other variables. What are then the criteria of its variability? Leibniz’s answer is that in the infinitesimal procedures the given evanescence quantity maintains, when it becomes zero, the ‘character’ of the quantity from which it derives. Thus, it can be ‘extensionally’ considered to be zero, but not ‘intensionally,’ because it keeps a relation with the quantity which was dissolved and from which it was derived. This is what Leibniz wrote to Grandi on the matter:

Interea infinite parva concipimus non ut nihila simpliciter et absolute, sed ut nihila respectiva (ut ipse bene notas), id est ut evanescentia quidem in nihilum retinentia tamen characterem ejus quod evanesci. (GM IV, p. 218).

When the evanescence is not simply close to zero, but it wholly ‘becomes’ zero we have a full resolution of the infinitesimal and, thus, its complete definition in the integral that expresses it. Being an evanescere a sort of process (or ‘activity’), it is important to see how the characterem of what vanishes is ‘retained’ (retentum), or ‘conserved,’ in its formal aspects when it has been entirely vanished (let us be reminded how this form of retention influenced Hegel on the concept of Aufhebung).

To define this particular context, Leibniz uses a term which was specific to his philosophical dictionary: that of ‘character.’ The infinitesimal sets down its quantitative aspect in to the zero, whilst it still keeps a sort of provenance mark: its ‘character.’ ‘Character’ is a sign, a symbol that expresses something else: it does not have a content in itself, but rather refers to a process of definition. Thus, ‘character’ means something ‘qualitative,’ which is ‘formal’ in Leibniz’s own terms, and represents the outcome of a whole process: it is, precisely, the pure form of its representation.

26 See A VI 4 C, p. 2768. On the concept of ‘character’ the references are, obviously, innumerable. I’ll only give a couple of them here from the writings of ’77 that seem to me to be fitting. In the first one he states: “Nam etsi characteres sint arbitrarii, eorum tamen usus et connexio habet quiddam quod non est arbitrarium, siclicet proportionem quandam inter characteres et res; et diversorum characterum easdem res exprimentium relationes inter se. Et haec proportio sive relatio est fundamentum veritatis” (Dialogus, A VI 4 A, p. 24). In the second, instead it is said that the characters ‘represent things,’ but do not identify themselves with ‘the thing itself’ (La vraie methode, A VI 4 A, p. 5).
4. Does singularity mean infinite?

On the one hand, the importance of the last statement lies in the fact that for a number of reasons it shifts the perspectives until now considered under a new logical (and also symbolical) dimension. On the other hand, this perspective refers again to the language of ‘substance,’ thus, to the issue of defining the singularity and its relation to accidental predicates.

Let us then go back to these problems and see if the mathematical considerations we have been focusing on can help to shed further light on this issue.

As we were saying, substance is, in its nature, a ‘totality’ and for this reason it is also ‘singular’ (remember that, according to Leibniz, two totalities can never overlap). Therefore, the predicate of singularity determines the manner of being of a substance in an ‘essential’ way.

Furthermore, a substance is also a collection of singular accidents. Singularity seems to express a kind of double level, because substance can be defined as something singular when it is considered as a whole, but at the same time, if we are to consider its inner-composition, every substance is somewhat structured by an infinite dimension of ‘singular’ determinations.

What relation subsists here between these two levels of singularity? This is where mathematical notions can be useful.27

Let us start by looking at the issues drawn in the Generales inquisitiones and let us move from there. First of all, Leibniz says that there are some ‘integral terms’ (terminus integralis, sive perfectus) to which ‘partial terms’ (partialis, sive imperfectus) refer to. Yet, before even reflecting on the relation between ‘integral’ and ‘partial,’ Leibniz focuses on the notion of ‘term,’ distinguishing different types of terms:

a) Simple primitive terms (such as ‘A’) - i.e., terms that are irresolvable in others, or taken as such.

b) Complex terms that are composed of two or more simple terms (as ‘AB’).

c) Simple derived terms that don’t emerge from a simple composition, but by using a simple particle – i.e., by using ‘primitive syncategorematic’ ones – as for instance ‘A in B.’

d) Complex derived terms that are composed of two or more derived terms.

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27 Leibniz himself stresses the importance of having a ‘sprinkling of maths’ to understand the relation between the ‘primitive predicates’ to the ‘complete concept’ that binds them. See MUGNAI. Introduzione, p. 198.
e) More complex cases that result from the combination of the previous points.

Leibniz imagines a combinatorial pattern structured by different kinds of integrals. Some are defined as ‘composed’ or ‘derived,’ to the extent that they can be divided into more elementary terms (‘partiales’). Others are ‘simple’ and cannot be divided any further. For this reason they are also defined as ‘purely integral terms.’

These purely integral terms are thus independent from any other combination of partial elements. It is rather the opposite: it is only the presence of some purely integral terms that allows the definition of the partial ones - because without the former the latter would prove to be pretty insignificant. Leibniz writes as follows:

Sed tamen ante partiales et particulias explicari debent illi integrales qui aut non resolvuntur, aut non nisi in integros. Et tales integrales a partialibus indipendentes utique esse necesse est, saltem generales, ut Terminus, Ens, nam his ipsi partiales indigent, ut transeunt in integrales. (A VI 4 A2, p. 741).

What are these “purely integral terms” Leibniz refers to? Which are those ‘simple primitive’ terms, that don’t derive from a further process of combination? The first simple integral thought of by Leibniz, the first element in the list, is the notion of ‘term’ itself. Putting it in mathematical terms, this is not in any way surprising. The concept of term represents a whole that is not itself composed (as a line, according to Leibniz, is not composed by a series of points), but it represents a higher-order function

What are then the other ‘simple primitive’ and purely integral terms? Here they are: Ens, Existens, Individuum, Ego.28

We are now in a position to sum up all of these complex lines of reasoning developed by Leibniz.

We first observed the close relation subsisting between the notions of ‘substance,’ ‘singularity’ and ‘concept’ and we were reminded, by the quoting of some very clear texts, that the complete concept expresses the substance’s essential singularity (conceptus autem completus est nota substantiae singularis).

Therefore, we have seen how both the singular accident and the whole substance could be defined only with reference to a ‘totality’, which involves a whole series of determinations.


We then looked upon the relations between ‘singularity’ and ‘totality,’ asking ourselves whether Leibniz’s argumentations were not indeed running along the line of a circular reasoning. On the same lines, the notion of singularity seemed to hide an ambiguous ‘double level’ of meanings, being essential for the definition of substance (‘Socrates’), as well for its inner accidents (‘snub nose’).

We can therefore outline four essential points:

1) Firstly, the accidents cannot be considered as partial elements of the substance, because the latter (being an ‘essence,’ an ‘existing,’ an ‘individual’ or an ‘I’) represents for Leibniz a ‘simple primitive,’ i.e. a ‘purely integral.’ That means that substance is not formed by ‘summing’ or ‘combining’ its predicates, in the same way as the continuum is not a summative of points.

2) Secondly, the relation between substance and accidents is similar to that between integrals and infinitesimals. The complete concept expresses this relation and, therefore, should itself be considered as a ‘simple primitive,’ a ‘singular term,’ or an integral. 29

3) Thirdly, the ‘double level’ played by the notion of singularity can be understood by taking into account the relations between the whole substance and its accidents, on one side, and the intra-predicative relations on the other – and they turn out to be very different. In the former case, the substance plays the role of an integral: it is a functional point of reference. In the latter, the function changes because it is each accident that from time to time becomes the singular term of reference, changing its relation to the other particular predicates. Therefore, it is not the definition of ‘singularity’ that changes, but the context in which it displays a function.

4) Lastly, if our assumptions are consistent, one should conclude that the semantic pair formed by the notions of ‘singular’ and ‘singularity’ does not refer to the cluster of notions defined by the lexicon of being ‘part,’ ‘partial’ or ‘infinitesimal’, but rather to that of ‘whole,’ ‘totality’ and ‘integral’ terms. In other words, singularity and totality are coextensive notions and for this reason, from a logical point of view, we do not have a circular reasoning: the concept of singularity is itself bound to that of a totality and each totality is, in turn, expression of a singularity.

29 See A VI 4, A2, p. 866.
5. Infinite versus Infinite: a comparison between different theoretical models

Let us, again, look upon and reconsider Hegel’s criticisms of Leibniz. The first question one should ask is if Hegel’s interpretation of Leibniz is not outdone by the latter’s logical-mathematical hinges on which the definition of singularity is tied.

At first glance, two options should be considered. On the one hand, Hegel makes, in general, a wide use of mathematical tools that, at least indirectly, derive from a Leibnitian's (and of course to a Newtonian's) context of analysis. It has often been said that Hegel’s theorization of infinity was widely influenced by seventeenth and eighteenth century mathematical thought. Therefore, Newton and, even more so, Leibniz were for Hegel rational paradigms that are not always explicit in his work. This can be said for the infinitesimal calculus, which serves as a concrete example of a ‘qualitative’ process of definition, but also for the setup of a logical-categorical system. Actually, in Hegel’s eyes, concepts such as those of ‘limit,’ ‘continuum,’ ‘moment,’ ‘integral,’ ‘evanescent quantity’ or ‘differential,’ define the logical map that forms the invisible backbone of his philosophical system.

Hegel might not be far from the truth when he says that the operative presence of mathematics of infinity is one of the deepest sources of the logical idea of self-movement. On the other hand, what Hegel does not accept of Leibniz is the ontology that supports his analysis on infinity and it is on this very ground that he redefines his thought.

We might therefore say that the paramount aspect here is the different interpretation of the concept of infinite given by Leibniz and Hegel. Equally decisive is also the semantic voltage that from here reverberate in the two philosopher’s apparatus of concepts.

As we have seen above, for Leibniz the essential element of his infinitesimal analysis consisted in the fact that infinitely small quantities would be considered as ‘evanescent quantities.’ And that means that they are not to be intended only as pure ‘zeros,’ but as ‘relative zeros,’ intending by this a dynamic interpretation of their vanishing point. Evanescent quantities tend to dissolve, but in the very instant of their nullification the character of the whole retrospective process is preserved.

For Hegel this kind of paradoxical determination represents the archetype of the very notion of ‘concept’ and in his logic (as can be seen clearly from JS II on) he carries out a sort of conceptual transliteration of these mathematical categories. ‘Maintaining’ the character of
what vanishes is decisive. The infinite is not the result of an intellectual determination, which is external to the thing itself, but it is rather the acknowledgement of the inconsistency of the concept of ‘determination’ itself as it was posed by the abstract intellect. Hence, the difference between ‘reason’ and ‘intellect.’

Infinite is not bound to ‘things in themselves,’ but to a process of self-experience, which enables the faculty of reason to acknowledge itself and to discover itself as the form of rationality. Thus, the notion of infinity, in Hegelian terms, can be considered as a kind of self-determining process, as the emerging of a conceptual ‘character’ that maintains its own negativity (the vanishing of intellectual determinations) as an aufgehoben negativity. Infinity holds for the ‘infinite acknowledgement’ of its own ‘vanishing’ and, thus, for the infinite acknowledgement of its own becoming a ‘concrete’ being. This is the point in which epistemology collapses into ontology, becoming ‘real.’ This kind of self-reflecting conceptuality always emerges in Hegel’s language from whatever perspective we may look at it.

It is here that Hegel conceptually distances his thought from that of Leibniz. According to the latter, the intentional feature of ‘zero’ is meaningful only within an algebraic process. In other words, it is significant only within a mathematical context, which poses an abstract relation of functionality. Outside this abstract context (remember that for Leibniz mathematical entities have just an ideal – and not concrete, or substantial - nature), formal intension would not be manageable in any way. Therefore, it would neither refer to anything nor would it maintain any particular meaning.

Unless one considers, as Hegel does, the entire logical-ontological relationship subsisting between the Concept itself and its specific determinations as a kind of super-relationship between an ‘integral’ and its ‘infinitesimals.’ Thus, turning the whole doctrine of the Concept into an enormous functional structure that involves the moments that define it. From this point of view, the single categorical contents, as they are exposed in the Logic, would play the role of a ‘conceptual algebra.’ And the ‘evanescent’ ‘characters’ of this structure should become the pawns of an internal (self-directed) game, in which the meaning of each category would be specified by a perpetual process of dissolution, functionally linked with a continuous re-emerging (as an aufgehoben) of the whole to which they refer to. To put

30 ‘Moment’ must be intended here as a mathematical term.
it in simple terms: the whole Concept would be considered as the process of its own dissolution: an evanescent structure that removes itself becoming Nature.

At the very beginning of this essay, we noticed how in the monadic individual the singularity “geht in seiner eigenen Totalität zu Grunde.” We can now ask ourselves, which singularity was intended there by Hegel. As we have seen above, the logical-mathematical structure that lies at the bottom of the Leibnitian notion of singularity does not seem to be undermined here, because, on the contrary, Hegel himself seems to point towards the same direction.

Instead, what is undermined is the ontological (let us say ‘substantial’) level on which the definition had been played by Leibniz. Hegel criticizes the infinite number of world-substances that populated his ontology, because he saw an incompatibility of references. Each Leibnitian substance is a conceptual world, thus qualitative, infinite and complete, but substances are infinite also in quantity and this multiplication of worlds (each of them absolutely self-related to itself and, consequently, ‘closed’ to the others) ended up creating a conflict between autonomy and dependence. Each world is completely autonomous, but at the same time each world ‘needs’ the presence of the others. And only a God can be invoked for playing the role of an architrave supporting the whole system.

Naturally, speaking of infinites, there is an intriguing overlapping of levels because Hegel, on the one hand, brings forth his own notion of ‘infinity’ as opposed to the ‘bad’ one performed by Leibniz; but on the other, his notion of infinite was largely drawn by Leibniz’s mathematical settings - and that indeed sounds indeed paradoxical, because for Leibniz the infinitesimal calculus was called upon to solve exactly the problem of an arithmetical ‘bad infinity.’

In this room of mirrors, where the definition of ‘true’ and ‘bad’ infinity seem to switch each others position and to constantly overlap, perhaps lies the key to grasping the meaning of an ‘ontology of singularity.’

Once again the decisive point involves a semantic stretch in the use of concepts, because an apparently similar lexical context implies, conversely, the output of diverging theoretical models. An important exemplification of this semantic shifting can be seen, for

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31 “Mea certe opinione nihil est in universitate creaturarum, quod ad perfectum suum conceptum non indiget alterius cujuscunque rei in rerum universitate conceptu, cum unaquaeque res influat in aliam quamcumque ita ut si ipsa sublata aut diversa esse fingeretur, omnia in mundo ab eius quae nunc sunt diversa sint futura.” GP II, p. 226.
instance, in the very notion of ‘contradiction.’ As we have seen, Hegel grants to this category the task of interpreting and explicating the concept of ‘evanescent quantity.’ On the other hand, Leibniz (as well as other mathematicians of his time) scarcely mentions the term ‘contradiction’ in his texts.

This might be surprising at first, if we consider that Leibniz was one of the major interpreters of the so-called ‘method of tangents.’ Nonetheless, if we only focus upon the mathematical point of view, appealing to ‘contradiction’ to explain the anomalous behaviour of such quantities seems unnecessary. Leaving aside the already reported consideration that for Leibniz mathematical entities pertain to the ideal sphere of essences, the contradiction welcomed by Hegel, of quantities that are both null and not-null, can be explained by considering the observer’s attitude towards them. The evanescent quantities are equivalent to zero when considered in their functional relation to whole numbers, and vice versa they are not-null when considered in their functional relation with differential terms. They do not change in themselves, but they do change according to the function they are displaying. From this point of view the role and the weight of contradiction dims and the whole topic is not any longer to be considered in ontological terms. Furthermore, as a backlash, it completely changes the theoretical asset from which Hegel poses the issue of singularity.

Yet, this is – seemingly - Leibniz’s point of view. Or better, this last statement explicitly suggests the difficulty of finding a connecting point between the two theoretical systems. Regarding the evanescent quantities Hegel, on his side, turns the matter around by putting at the core of his thought the contradiction that exists in the very concept of ‘evanescent’ and he allows himself to use the speculative results of the mathematics of infinity.

In conclusion, the problem of the infinite represents one of the major challenges of Hegel’s thought and it constitutes both a starting as well as a finishing point of his speculative philosophical language. However, the focal point of his interpretation of Leibniz (or at least of his mathematical categories) is represented by his peculiar and somewhat autonomous conception of ‘contradiction.’ It is precisely this latter that intervenes as a semantic medium that somehow modifies from within the very concept of mathematical infinite and transforms it into a conceptual melting point in which logic and ontology come to be one. In this subtle, almost imperceptible way, the whole matter semantically slides and yet it apparently remains the same under the outward appearance of a common lexicon.

32 See PASINI. La Philosophie des mathématiques, p. 956.
This is probably the reason that explains why the comparison between Leibniz and Hegel, when considering the history of their philosophical interpretations, is fraught with theoretical ambiguity.

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